Physics 200A **Theoretical Mechanics** **Fall 2015**

Problem Set II: Due Tuesday, October 13, 2015

"I do not like to be dunned and teased by foreigners about mathematical things."
– Isaac Newton

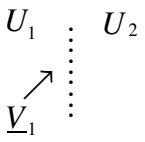
1.) Consider the nonlinear string problem, as developed in class.

a.) Derive the Lagrangian and Lagrangian equations of motion. What is the pragmatic criterion for the reduction of these to a wave equation?

b.) Derive the string Hamiltonian and the Hamiltonian equations of motion.

c.) Derive an energy theorem for the linear string. Discuss its correspondence to the Poynting theorem. What are the wave energy density and energy flux density?

2.) Consider a particle moving in:



a.) V_1 is the initial velocity. How does the direction change? What is V_2?

b.) Find the ratio of times in the same path for particles with different masses but the same U.

c.) Find the ratio of times in the same path for particles with the same mass but moving different potentials U_1, U_2, where $U_2/U_1 = c$, a constant.

d.) What problem in optics does this problem resemble?

3.) FW: 6.4

4.) FW: 6.2

5.) FW: 6.3 (It's assumed you are familiar with the heavy symmetric top from undergrad mechanics.)

6a.) For a particle of mass m moving thru a potential $U(x)$, the time-independent Schrodinger equation is:

$$-\left(\frac{\hbar^2}{2m}\right)\frac{d^2\Psi}{dx^2}+U\Psi=E\Psi\,.$$

$\Psi(x)$ is the wave function and E is the energy. $\Psi^{\star}$ is the complex conjugate to Ψ.

Show that the Schrodinger equation is the Lagrange Equation for:

$$L=\frac{-\hbar^2}{2m}\left|\frac{d\Psi}{\partial x}\right|^2-\Psi^*(U-E)\Psi\,.$$

b.) In this spirit, how would you go about showing conservation of probability?

7a.) In the case $L=L(q,\,\dot{q},\ddot{q},t)$, derive the corresponding equation of motion.

b.) How does one formulate energy conservation for such a system? Begin from the requirement of homogeneity in time.

8.) What is the shape of the hill down which you could ski without friction such that you would descend in the least time? Assume energy is conserved and that the only force acting is that of gravity.